Marks

Q.2 (a) $x^2 - 2x + x - 2 \geqq 0$ **1.0**

$x(x-2) + 1(x-2) \geqq 0$

$(x - 2)(x + 1) \geqq 0$ **1.0**

If $(x - 2)(x + 1) = 0$ then $x = 2$ or $x = -1$ **1.0**

If $(x - 2)(x + 1) > 0$ or $x > 2$ and $x > -1$ then $x > 2$ or **1.0**

If $(x - 2)(x + 1) > 0$ or $x < 2$ and $x < -1$ then $x < -1$ **1.0**

Solution: $x \leqq -1$ or $x \geqq 2$. **1.0**

(b) Here, a = 5 ½ , d = 4 – 5 ½ = 4 – 11/ 2 = 8 – 11 / 2 = - 3/2 & n = 12

Formula : S_n = n / 2 { 2a + (n – 1) d } **1.0**

= 12 / 2 { 2 (11/ 2) + (12 – 1) (- 3/ 2) }

= 6 { 2 × 11/ 2 + 11 × - 3/ 2 }

= 6 { 11 – 33 / 2 }

= 6 { 22 – 33 / 2 }

= 6 { - 11/ 2 }

= 3 × - 11 **1.0**

= - 33 Ans **1.0**

(c) Data; We have R = 5000, n =10 and i = 0.06 **1.5**

$$S = R\,\frac{(1+i)^n - 1}{i}$$ **1.0**

$$S = 5000\,\frac{(1+0.06)^{10} - 1}{0.06}$$ **1.5**

$$S = 5000\,\frac{(1.06)^{10} - 1}{0.06}$$

$$S = 5000\,\frac{1.790847697 - 1}{0.06}$$ **1.0**

$$S = 5000\,\frac{0.790847696}{0.06}$$

$$S = 5000(13.18079494)$$

S = 65, 903. 97471 **1.0**

ALTERNATE METHOD:

(By using Table) :

Data :

n = 10 years , i = 6% , R = 5000. **1.5**

$S = Rs_{\overline{n|}}\ I$ **0.5**

S = 5000$s_{\overline{10|}}$ **0.06** **1.5**

From table value:

$s_{\overline{10|}}$ 0.06 = 13.18079 **1.0**

S = 5000(13.18079) **0.5**

S= 65, 903. 95 **1.0**

(d) (i)

$$\int (x^3 + 4)^5 x^2 \, dx$$

$$= \frac{3}{3} \int (x^3 + 4)^5 x^2 dx$$

$$= \frac{1}{3} \int (x^3 + 4)^5 3x^2 dx$$ **1.0**

(Using the rule: $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + C$ *when n ≠ -1)* **0.5**

$$= \frac{1}{3}\left[\frac{(x^3 + 4)^{5+1}}{5+1}\right] + C$$

$$= \frac{1}{3} \times \frac{1}{6} (x^3 + 4)^6 + C$$ **0.5**

$$= \frac{1}{18} (x^3 + 4)^6 + C$$ **1.0**

(ii)

$f(x,y) = 3x^2 - 6xy + 4y^3$

$f_x = 6x - 6y$ **1.0**

$\mathbf{f_{xy} = -6}$ **1.0**

Q.3 (a) Here , the matrix form is

$$|A| = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 5 & 8 \end{vmatrix}$$

$$= 2\begin{vmatrix} 2 & 3 \\ 5 & 8 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ 4 & 8 \end{vmatrix} + 4\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$$

$|A| = 2(16 - 15) - 3(8 - 12) + 4(5 - 8)$

$|A| = 2(1) - 3(-4) + 4(-3)$

$|A| = 2 + 12 - 12$

$\mathbf{|A| = 2}$ **1.0**

Since $|A| = 2$, therefore the above matrix is non-singular, the solution does exits. **1.0**

Let $$|A_1| = \begin{vmatrix} 20 & 3 & 4 \\ 14 & 2 & 3 \\ 38 & 5 & 8 \end{vmatrix}$$

$|A_1| = 20(16 - 15) - 3(112 - 114) + 4(70 - 76)$

$= 20(1) - 3(-2) + 4(-6)$

$= 20 + 6 - 24$

$\mathbf{= 2}$ **1.0**

Let $$|A_2| = \begin{vmatrix} 2 & 20 & 4 \\ 1 & 14 & 3 \\ 4 & 38 & 8 \end{vmatrix}$$

$|A_2| = 2(112 - 114) - 20(8 - 12) + 4(38 - 56)$

$= 2(-2) - 20(-4) + 4(-18)$

$= -4 + 80 - 72$

$= 76 - 72$

$= 4$ **1.0**

Let $|A_3|$ **=** $\begin{vmatrix} 2 & 3 & 20 \\ 1 & 2 & 14 \\ 4 & 5 & 38 \end{vmatrix}$

$|A_3|$ **=** $2(76 - 70) - 3(38 - 56) + 20(5 - 8)$

$= 2(6) - 3(-18) + 20(-3)$

$= 12 + 54 - 60$

$= 66 - 60$

= 6 **1.0**

Now, $\mathbf{X_1} = |A_1| / |A| = 2/2 = \mathbf{1}$ **1.0**

$\mathbf{X_2} = |A_2| / |A| = 4/2 = \mathbf{2}$ **1.0**

and $\mathbf{X_3} = |A_3| / |A| = 6/2 = \mathbf{3}$ **1.0**

Therefore, the solution is

$\mathbf{X_1 = 1,\ X_2 = 2}$ **and** $\mathbf{X_3 = 3}$

(b) As x_1 = Number of litres sold of regular

x_2 = Number of litres sold of premium

(i) Total revenue from the sale of the two different grades of petrol

$R = f(x_1, x_2) = p_1x_1 + p_2x_2$ **0.5**

OR $R = f(x_1, x_2) = 70x_1 + 80x_2$ **1.5**

(ii) Total cost = total fixed cost + total variable cost **0.5**

Here , total fixed cost = 0 **0.5**

$C = f(x_1, x_2) = 55x_1 + 70x_2$ **1.0**

(iii) Total profit = total revenue – total cost

$P(x_1, x_2) = R(x_1, x_2) - C(x_1, x_2)$

$P(x_1, x_2) = (70x_1 + 80x_2) - (55x_1 + 70x_2)$

$P(x_1, x_2) = 70x_1 + 80x_2 - 55x_1 - 70x_2$

$= 15x_1 + 10x_2$ **1.0**

(iv) $P(x_1, x_2) = 15x_1 + 10x_2$

$P(80{,}000, 30{,}000) = 15(80{,}000) + 10(30{,}000)$

$= 1{,}200{,}000 + 300{,}000$

$=$ Rs.1,500,000 **1.0**

(c) (i)

$h = f(t) = 3.5\,t^3$, $0 \le t \le 30$ sec

Average velocity during the time interval $0 \le t \le 15$ sec

$$\frac{\Delta h}{\Delta t} = \frac{f(15) - f(0)}{15 - 0}$$ **0.5**

$$= \frac{3.5(15)^3 - 3.5(0)^3}{15}$$ **0.5**

$$= \frac{3.5(3375) - 3.5(0)}{15}$$

$$= \frac{11{,}812.5}{15}$$ **1.0**

Their for:

$$\frac{\Delta h}{\Delta t} = 787.5 \text{ ft / sec}$$ **1.0**

(ii) To find Instantaneous velocity differentiating w.r.t on both sides

$$\frac{dh}{dt} = \frac{d\,(3.5\,t^3)}{dt}$$

$$\frac{dh}{dt} = 3\,(3.5\,t^{3-1}) = 3\,(3.5\,t^2) = 10.5\,t^2$$ **1.0**

At t =15 sec

$$\frac{dh}{dt} = 10.5(15)^2 = \mathbf{2{,}362.5\ ft\ /\ sec}$$ **1.0**

At t =20

$$\frac{dh}{dt} = (10.5\,(20)^2) = (10.5)(400)$$

$$\frac{dh}{dt} = \mathbf{4200\ ft\ /\ sec}$$ **1.0**

Q.4 (a)

Year	Annual Values	5-Yearly Total	Trend	
1987	240	-	-	
1988	270	-	-	
1989	238	1257	251	**0.25**
1990	252	1267	253	**0.25**
1991	257	1280	256	**0.25**
1992	250	1312	262	**0.25**
1993	283	1328	266	**0.25**
1994	270	1351	270	**0.25**
1995	268	1385	277	**0.25**
1996	280	1412	282	**0.25**
1997	284	1442	288	**0.25**
1998	310	1477	295	**0.25**
1999	300	1495	299	**0.25**
2000	303	1524	305	**0.25**
2001	298	1554	311	**0.25**
2002	313	1563	313	**0.25**
2003	340	1589	318	**0.25**
2004	309	1624	325	**0.25**
2005	329	1638	328	**0.25**
2006	333	1683	337	**0.25**
2007	327	1718	344	**0.25**
2008	385	1732	346	**0.25**
2009	344	-	-	
2010	343	-	-	

(b) (i)

$n = 6 \qquad r = 2$

$^{n}p_{r} = {}^{6}p_{2}$

$= \dfrac{n!}{(n-r)!}$ **0.5**

$= \dfrac{6!}{4!}$

$= \dfrac{6 \times 5 \times 4!}{4!} = 6 \times 5$ **0.5**

$=$ **30** **1.0**

(ii)

$p = 1/6 \qquad q = 1 - \dfrac{1}{6} = \dfrac{5}{6} \qquad n = 6 \qquad x = 4$ **0.5**

Required probability $= b(x;k;p)$

$= \binom{k}{x}(p)^{x}(q)^{k-x}$ **0.5**

$= b(4;6;1/6)$

$= \binom{6}{4}\left(\dfrac{1}{6}\right)^{4}\left(\dfrac{5}{6}\right)^{6-4} = \dfrac{6!}{4!(6-4)!}\left(\dfrac{1}{6}\right)^{4}\left(\dfrac{5}{6}\right)^{2}$

$= \dfrac{6 \times 5 \times 4!}{4!2!} \times \dfrac{1}{1296} \times \dfrac{25}{36} = \dfrac{6 \times 5 \times 4!}{4! \times 2 \times 1} \times \dfrac{1}{1296} \times \dfrac{25}{36}$

$= 15 \times \dfrac{1}{1296} \times \dfrac{25}{36} = \dfrac{375}{46656}$ **1.0**

$=$ **0.008** **1.0**

Q.5 (a) This is a one-tail test as we are only interested in whether the mean weight is less than the nominal or not.

H_0: mean = 500 grams **0.5**

H_1: mean < 500 grams **0.5**

Degrees of freedom = n – 1 = 16 – 1 = 15 **0.5**

Data:

n = 16,

$\bar{x}$ = 495 grams,

s = 15,

Level of Significance = 0.05

Standard error of the mean = $\frac{s}{\sqrt{n}} = \frac{15}{\sqrt{16}} = 3.75$ **0.5**

Therefore t = $\frac{\bar{x}-\mu}{s_{\bar{x}}} = \left|\frac{495-500}{3.75}\right| = 1.33$ **1.0**

Tabulated value of "t" is 1.753 **1.0**

Thus as the calculated *t* score, 1.33, is less than 1.753 we can accept the null hypothesis and reject the alternative hypothesis at the 5% level. **1.0**

(b) (i) Here n = 7 and r = 5

$^nC_r = \frac{n!}{r!(n-r)!}$ **0.5**

$= \frac{7!}{5!(7-5)!}$

$= \frac{7!}{5! \times 2!}$ **0.5**

$= \frac{7 \times 6 \times 5!}{5! \times 2 \times 1}$

= 7 x 3

= 21 **1.0**

(ii) A : 1st card is an ace

J : 2nd card is a jack

Now $P(A)$ = **4/52** **0.5**

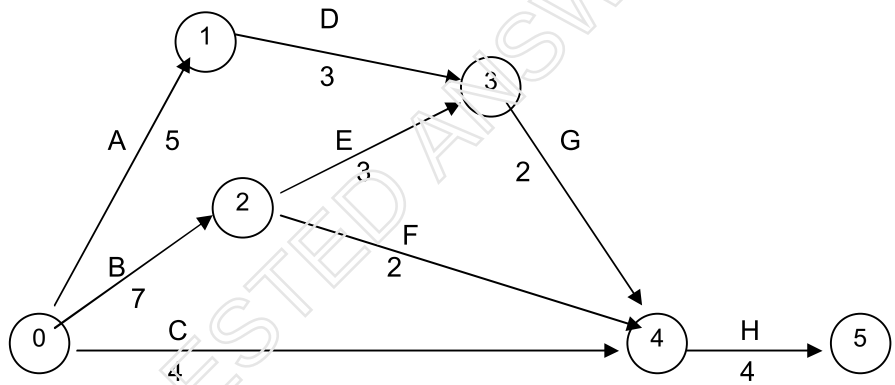
$P(J/A)$ = **4/51** **0.5**

Hence

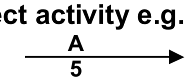
$P(A \cap J) = P(A)\, P(J/A) = 4/52 \times 4/51 = 2/26 \times 4/51$ **1.0**

$P(A \cap J)$ = **8/1326** **1.0**

Q.6 (a)



(0.5 mark for drawing of each correct activity e.g. 0.5 mark for drawing of:



(b) **BEGH (7+3+2+4)** **1.0**

= 16 Weeks **1.0**

(c) EST and LST:

Activity	EST	LST	
A	0	0	**0.25x2**
B	0	0	**0.25x2**
C	0	0	**0.25x2**
D	5	7	**0.25x2**
E	7	7	**0.25x2**
F	7	7	**0.25x2**
G	10	10	**0.25x2**
H	12	12	**0.25x2**

Q.7 (a) Let x_1: Number of kg of ingredient of type A

x_2: Number of kg of ingredient of type B

Minimize	$C = 12x_1 + 8x_2$		**0.5**
Subject to	$x_1 + x_2 \geq 100$	(Weight Constraint)	**0.5**
	$0.30x_1 + 0.10x_2 \geq 15$	(Nitrogen Constraint)	**0.5**
	$0.10x_1 + 0.05x_2 \geq 8$	(Phosphate Constraint)	**0.5**
	$0.20x_1 + 0.40x_2 \geq 25$	(Bone Meal Constraint)	**0.5**
	$x_1, x_2 \geq 0$	(Non- Negativity Constraint)	**0.5**

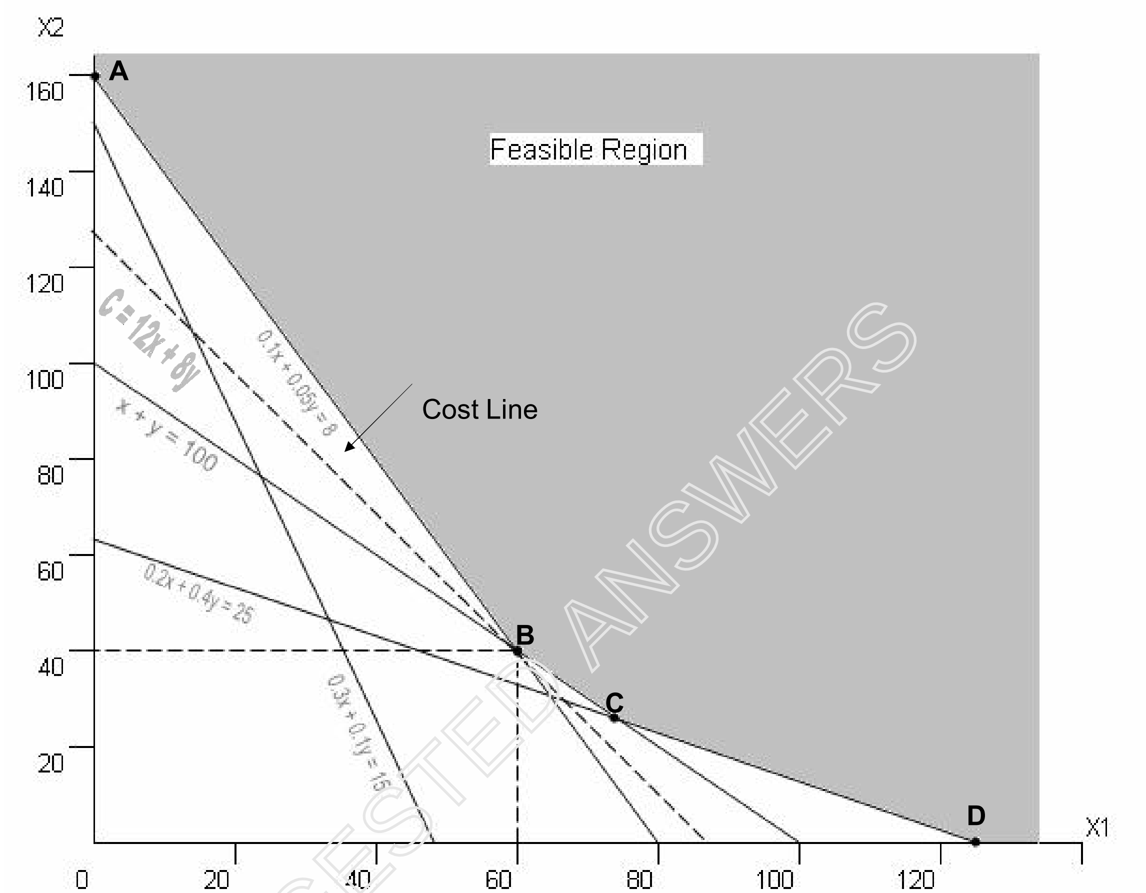
(b) ***Corner Point Soluton*** **:**

The corner points of the area of feasible solution in the graph below is given as:

(x_1,x_2)	**$C = 12x_1 + 8x_2$**	
(0 , 160)	1,280	**0.25x2**
(60 , 40)	**1,040**	**0.25x2**
(125 , 0)	1,500	**0.25x2**
(75 , 25)	1,100	**0.25x2**

Hence the total cost is minimized at **x_1 = 60** & **x_2 = 40** **0.5**

The Total Minimum Cost is **Rs. 1040** **0.5**



Marks Distribution:

0.5 mark each for drawing of 5 correct lines (0.5 x 5)	**= 2.5**
Identification of four corner points (0.25 mark x 4)	**= 1.0**
Identification of feasible region	**= 0.5**
	4.0

THE END